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MOTION OF A GAS IN A CYCLONE HEAT EXCHANGER

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A semi-empirical theory of turbulence is used to obtain relations for calculation of the field of gas velocity in the flow core and boundary layer in a cyclone chamber. To close the system of Navier-Stokes equations, the apparent shear stress is represented in the form of the gradient dependence in circulation. The relations for calculating the tangential component of velocity were derived using experimental data on the qualitative character of the distribution of apparent shear stress over the radius of the cyclone.

In order to devise methods of calculating heat- and mass-transfer processes between a gas and particles in cyclone heat exchangers, it is necessary to thoroughly examine the aerodynamic structure of the twisted disperse flow — including the velocity field of the gas in the cyclone. Most investigations of the aerodynamics of cyclone chambers have been experimental studies of the distribution of the components of gas velocity in the core of the flow. Different empirical relations have been proposed for calculation mainly of the tangential component of velocity. There has been little study of the boundary-layer gas flow in cyclones, even though the dispersed material moves mainly in this region in cyclone chambers with "dry" walls. In the present study, we attempt to use a semi-empirical theory of turbulence to obtain relations for calculating the velocity field of the gas flow throughout the volume of a cyclone chamber.

We will represent the running parameters of the flow as consisting of the time-averaged radial u , axial w , and tangential v components of velocity and the fluctuation components u' , w' , v' . Then the Navier-Stokes equations are augmented by expressions for the Reynolds stresses. We will write this equation in cylindrical coordinates in the tangential direction, taking into account the symmetry of the twisted flow relative to the axis of the cyclone:

$$u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial x} + \frac{vu}{r} = v \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial v r}{\partial r} \right) + \frac{\partial^2 v}{\partial x^2} \right] + \frac{1}{\rho} \left[\frac{\partial \tau_{r\varphi}}{\partial r} + \frac{2}{r} \tau_{r\varphi} + \frac{\partial \tau_{x\varphi}}{\partial x} \right], \quad (1)$$

where $\tau_{r\varphi} = -\rho u'v'$, $\tau_x = -\rho v'w'$ are components of the turbulent shear stress.

Here, the continuity equation will have the form

$$\frac{1}{r} \frac{\partial ur}{\partial r} + \frac{\partial w}{\partial x} = 0. \quad (2)$$

To close the system of Navier-Stokes equations in the theory of turbulent motion [1], empirical relations are introduced to link the apparent shear stress with the time-averaged velocities. For $\tau_{r\varphi}$, this connection is usually expressed by one of two methods:

1) by generalization of the Karman similarity hypothesis to curvilinear flows

$$\tau_{r\varphi} = \rho \nu_t r \frac{d}{dr} \left(\frac{v}{r} \right); \quad (3)$$

2) by making use of Prandtl's theory of momentum transfer

$$\tau_{r\varphi} = \rho v_{wa} \frac{1}{r} \frac{d}{dr} (vr). \quad (4)$$

The use of Eq. (3) with $v_t = \text{const}$ yields good results in the study of slightly twisted turbulent jets [2, 3]. However, it was shown in [4] that the use of this expression for a plane vortex leads to paradoxical results. First of all, it suggests that no energy is dissipated in the rigid rotation of the gas (in the axial region of the twisted flow). This could be possible only due to the appreciable turbulence in this region. Secondly, the use of Eq. (3) in the given case also indicates that energy is dissipated during nonvortical motion.

The velocity profiles of a swirled gas flow in a cyclone were calculated using an expression associated with (3)

$$\tau_{r\varphi} = \rho l^2 \left[r \frac{d}{dr} \left(\frac{v}{r} \right) \right]^2, \quad (5)$$

where l is the mixing length. Here, we also made use of the approximation of the tangential component of velocity proposed in [4]. It was found that the resulting distributions of the axial and radial velocities deviated from the experimental data (at $\eta \rightarrow 1$, $u \rightarrow \infty$, $w \rightarrow -\infty$). Here, the change in v_t' over the radius of the cyclone is of an alternating character. The quantity $\tau_{r\varphi}$ takes a value of zero in the transitional region between quasi-potential and quasi-rigid rotation and increases by 1-2 orders in the transition from the quasi-rigid rotation zone to the quasi-potential rotation zone. Such a character of distribution of v_t and $\tau_{r\varphi}$ is unlikely. Thus, there is no foundation for the use of Eq. (5) to close the system of Navier-Stokes equations. In [4], the shear stress in the turbulent core of a rotating flow was represented in the form of the gradient dependence on circulation $\Gamma = vr$

$$\tau_{r\varphi} = \kappa^2 \rho \left[\frac{\partial (vr)}{\partial r} \right]^2 = \rho l^2 \left[\frac{1}{r} \frac{\partial (vr)}{\partial r} \right]^2, \quad (6)$$

where κ is a quantity which characterizes the turbulence structure of the twisted flow and $l = \kappa r$ for the flow core.

In contrast to (3) and (5), it follows from (6) that $\tau_{r\varphi} = 0$ for a plane vortex with potential rotation of the gas. With rigid rotation, $\tau_{r\varphi} = 4\rho v_m \kappa^2 (r/r_m)^2$, i.e., in this region the radial distribution of shear stress is described by a quadratic law. The values of $\tau_{r\varphi}$ calculated with (6) agree with the experimental data in [4].

As regards the components of the tensor of shear stress τ_x , by analogy with rectilinear motion of the gas we can write [5]

$$\tau_{x\varphi} = \rho v_{x\varphi} \left(\frac{\partial v}{\partial x} \right). \quad (7)$$

In the core of a twisted flow, it can be assumed with a high degree of confidence that $v \ll v_t$ and $v \ll v_{x\varphi}$. Inserting (4) and (7) into (1), we obtain

$$\frac{u}{r} \frac{\partial (vr)}{\partial r} + w \frac{\partial v}{\partial x} = \frac{1}{\rho r^2} \frac{\partial (\tau_{r\varphi} r^2)}{\partial r} + \frac{\partial}{\partial x} \left(v_{x\varphi} \frac{\partial v}{\partial x} \right). \quad (8)$$

In the general case, the component of the shear-stress tensor $\tau_{r\varphi}$ is a function of the radial r and axial x coordinates. This function can be represented in the form

$$\tau_{r\varphi} = \tau_{wa} \varphi^2(x) f^2(r),$$

then from (6) we obtain

$$\frac{\partial v \eta}{\partial \eta} = \frac{R_c \eta}{l} \sqrt{\frac{\tau_{wa}}{\rho}} \varphi(\xi) f(\eta), \quad (9)$$

where $\eta = r/R_c$; $\xi = x/L_c$.

Proceeding on the basis of experimental data on the character of the distribution of $\tau_{r\varphi}$ [6], we can represent the function $f(\eta)$ for the core of the flow in the form

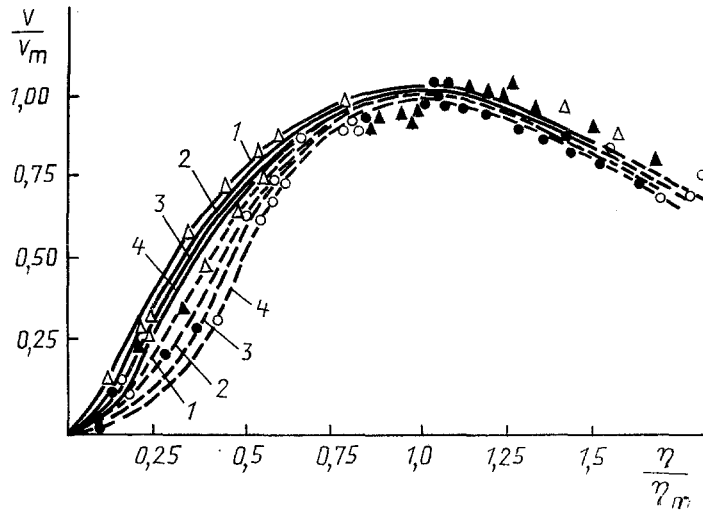


Fig. 1. Comparison of theoretical profiles of tangential velocity for the flow core and experimental data from [4]: 1) $m = 2.07$; 2) 2.25; 3) 2.46; 4) 2.96; solid curves show results calculated from (12); dashed curves show results obtained from the Shtym formula [4].

$$f(\eta) = \eta^n (\eta_{co} - \eta)^2. \quad (10)$$

Since the rotation of the gas in the axial region is similar to the rotation of a rigid body, we can propose as a first approximation that the mixing length $l = \kappa r$ in this region. Then keeping in mind (10) and the fact that $v = 0$ at $\eta = 0$, we obtain the following for tangential velocity from (9)

$$v = \frac{\varphi(\xi)}{\kappa} \sqrt{\frac{\tau_{wa}}{\rho}} \eta^n \left[\frac{\eta_{wa}^2}{n+1} - \frac{2\eta_{co}\eta}{n+2} + \frac{\eta^2}{n+3} \right]. \quad (11)$$

Experiments show [4] that as a first approximation we can take $\varphi(\xi) = 1$ away from the end regions of the cyclone. Considering that $v = v_m$ and $\eta = \eta_m$, we can easily use (11) to determine the value of the parameter n :

$$n = \sqrt[3]{-\frac{q}{2} + \sqrt{Q}} + \sqrt[3]{-\frac{q}{2} - \sqrt{Q}} - \frac{5}{3},$$

where

$$q = -\frac{a^2 + 16a + 10}{13.5(a-1)^2}; \quad Q = \left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2;$$

$$p = \frac{7-a}{3(a-1)}; \quad a = \frac{\eta_m}{\eta_{co}}.$$

Then finally

$$\bar{v} = \frac{v}{v_m} = \left(\frac{\eta}{\eta_m}\right)^n \frac{\frac{\eta_{co}^2}{n+1} - \frac{2\eta_{co}\eta_m}{n+2} + \frac{\eta^2}{n+3}}{\frac{\eta_{co}^2}{n+1} - \frac{2\eta_{co}\eta}{n+2} + \frac{\eta_m^2}{n+3}}. \quad (12)$$

The characteristic radii of the flow η_{co} and η_m and the maximum value of tangential velocity v_m can be determined by using the empirical relations proposed in [4]. Knowing the value of these parameters, we can readily determine the quantity $1/\kappa\sqrt{\tau_{wa}/\rho}$. Good agreement was obtained between Eq. (12) and the empirical relation constructed by A. N. Shtym for tangential velocity (Fig. 1). The figure also shows that Eq. (12) agrees well with the experimental data in [5]. Using (12), we obtain the following formula from (8) to determine the radial component of the gas in the flow core

$$\bar{u} = \frac{u}{v_m} = 2\kappa^2 \left(\frac{\eta}{\eta_m} \right)^n \frac{(\eta_{co} - \eta) [\eta_{co}(n+1) - \eta(n+3)]}{\left[\frac{\eta_m^2}{n+1} - \frac{2\eta_{co}\eta_m}{n+2} + \frac{\eta_m^2}{n+3} \right]}, \quad (13)$$

while with allowance for (13) we obtain the below expression from (2) to determine the axial velocity for the gas at $0 \leq \eta \leq \eta_{co}$

$$\bar{w} = \frac{w}{v_m} = -2 \frac{L_c}{R_c} \kappa^2 \frac{\eta^{n-1} [(n+3)^2 \eta^2 - 2\eta_{co}(n+2) + \eta_{co}^2 (n+1)^2]}{\eta_m^n \left[\frac{\eta_{co}^2}{n+1} - \frac{2\eta_{co}\eta_m}{n+2} + \frac{\eta_m^2}{n+3} \right]} \times \xi + C_1(\eta). \quad (14)$$

The aerodynamics of the swirled flow in the wall region of the chamber, of thickness $\delta = R_c(1 - \eta_{co})$, differs from the aerodynamics of the core. We adopt the mixing length $l = K(R_c - r)$ within the boundary layer and we designate the coordinate of the boundary of the laminar sublayer as η_0 . Then in accordance with the chief postulates of the semi-empirical turbulence theory we can write [7]:

$$\eta_m \leq \eta < \eta_0, \quad \frac{\tau}{\rho v_m^2} = \frac{K^2(1-\eta)^2}{\eta^2} \left[\frac{\partial(\bar{v}\eta)}{\partial\eta} \right]^2, \quad (15)$$

$$\eta_0 < \eta \leq 1, \quad \frac{\tau}{\rho v_m^2} = \frac{v}{R_c v_m} \left[\frac{\partial(\bar{v}\eta)}{\partial\eta} \right], \quad (16)$$

$$\bar{v}_{\eta=\eta_0+0} = \bar{v}_{\eta=\eta_0-0}; \quad \tau_{\eta=\eta_0+0} = \tau_{\eta=\eta_0-0}; \quad (17)$$

$$\left(\frac{\partial v \eta}{\partial \eta} \right)_{\eta=\eta_0+0} = K_1 \left(\frac{\partial v \eta}{\partial \eta} \right)_{\eta=\eta_0-0}. \quad (18)$$

We represent the function $\tau = \tau(\eta)$ in the form

$$\frac{\tau}{\tau_{wa}} = \left(\frac{\eta - \eta_{co}}{1 - \eta_{co}} \right)^4. \quad (19)$$

Having integrated (15) and (16) with allowance for (19), we obtain the distribution of tangential velocity in the wall region of the cyclone: at $\eta_{co} \leq \eta \leq \eta_0$

$$\frac{v}{v_{co}} = \frac{\eta_{co}}{\eta} \left\{ 1 + \frac{1}{\eta_{co} K v_{co}} \sqrt{\frac{\tau_{wa}}{\rho}} \left[\ln \frac{1-\eta}{1-\eta_{co}} + \frac{1}{3} \left(\frac{\eta - \eta_{co}}{1 - \eta_{co}} \right)^3 + \frac{3}{2} \left(\frac{\eta - \eta_{co}}{1 - \eta_{co}} \right) \right] \right\}, \quad (20)$$

$$v = v_{co} \quad \text{at } \eta = \eta_{co}$$

while at $\eta_0 \leq \eta \leq 1$ (having in mind that the thickness of the laminar sublayer is small enough so that we can take $\eta_0 \approx 1$):

$$\bar{v} = \frac{\tau_{wa}}{\rho} \frac{R_c}{v v_m} (1 - \eta). \quad (21)$$

Having used Eqs. (15-16) and condition (19), we find the thickness of the laminar sublayer

$$\eta_0 = 1 - \frac{K_1}{K} \frac{1 - \eta_{co}}{C \text{Re}_{co}}, \quad (22)$$

where

$$C = \frac{1}{v_{co}} \sqrt{\frac{\tau_{wa}}{\rho}}; \quad \text{Re}_{co} = \frac{v_{co} R_c (1 - \eta_{co})}{\nu}$$

We believe that there is a certain analogy between the rotation of the gas in the boundary region of the cyclone and the motion of a gas in a turbulent boundary layer in the case of flow past a plate. Thus, we can write [7]

$$C = \frac{1}{v_{co}} \sqrt{\frac{\tau_{wa}}{\rho}} = C_0 (\text{Re}_{co})^{-m} = C_0 \left(\frac{1 - \eta_{co}}{2} \right)^{-m} \left(\frac{v_{co}}{v_{in}} \right)^{-m} \left(\frac{v_{in} 2 R_c}{\nu} \right)^{-m},$$

where for flow about the plate $C_0 = 0.24$, $m = 0.125$, $K = 0.4$. With allowance for this, we finally obtain the following to determine the field of the tangential component of gas velocity in the boundary region of the cyclone

at $\eta_{co} \leq \eta \leq \eta_0$

$$\frac{v}{v_{co}} = \frac{\eta_{co}}{\eta} \left\{ 1 + \frac{C_0 Re_{co}^{-m}}{\eta K} \left[\ln \frac{1-\eta}{1-\eta_{co}} + \frac{1}{3} \left(\frac{\eta-\eta_{co}}{1-\eta_{co}} \right)^3 + \frac{3}{2} \left(\frac{\eta-\eta_{co}}{1-\eta_{co}} \right) \right] \right\}, \quad (23)$$

at $\eta_0 \leq \eta \leq 1$

$$\frac{v}{v_{co}} = C_0^2 Re_{co}^{1-2m} \frac{1-\eta}{1-\eta_{co}}, \quad (24)$$

where v_{co} is determined from (12) with $\eta = \eta_{co}$.

The values of the parameters C_0 , m , and K were determined by comparing distributions of tangential gas velocity in the cyclone boundary region calculated from (23) and established experimentally [8]. Tangential velocity was measured in [8] in a cyclone $R_c = 0.07$ m with $Re_{co} = 8100$. Here, the thickness of the boundary layer changed within the range 8-25 mm, i.e., $0.65 \leq \eta_{co} \leq 0.885$. It is evident from Fig. 2 that at $C_0 = 0.231$, $m = 0.125$, and $K = 0.275$, the theoretical curves $\eta_{co} = 0.65$ and 0.885 completely envelope the experimental points. For comparison, Fig. 2 shows curves of velocity distribution in the boundary layer that were proposed in [1] for a plate (curve 4) and in [9] for a cyclone (curve 3).

Using (23), under the condition that $\partial v / \partial x = 0$ we can use (1-2) to determine the radial and axial components of gas velocity in the boundary layer:

$$\begin{aligned} \frac{u}{v_m} &= 2KC_0 Re_{co}^{-m} \left(\frac{\eta_{co}}{\eta_m} \right)^n \times \\ &\times \frac{\left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \right] (1-\eta)(\eta-\eta_{co})(3\eta-\eta_{co})}{\left[\frac{1}{n+1} - \frac{2\eta_m}{(n+2)\eta_{co}} + \frac{1}{n+3} \left(\frac{\eta_m}{\eta_{co}} \right)^2 \right] \eta (1-\eta_{co})^2}, \end{aligned} \quad (25)$$

$$\begin{aligned} \frac{w}{v_m} &= -2 \frac{L_c}{R_c} KC_0 Re_{co}^{-m} \left(\frac{\eta_{co}}{\eta_m} \right)^n \times \\ &\times \frac{\left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \right] (6\eta - 4\eta_{co} - 9\eta^2 + 8\eta\eta_{co} - \eta_{co})\xi}{\left[\frac{1}{n+1} - \frac{2}{n+2} \frac{\eta_m}{\eta_{co}} + \frac{1}{n+3} \left(\frac{\eta_m}{\eta_{co}} \right)^2 \right] \eta (1-\eta_{co})^2} + C_2(\eta). \end{aligned} \quad (26)$$

It follows from the condition for the joining of velocity profiles (14) and (26) at $\eta = \eta_{co}$ that $C_1(\eta) = C_2(\eta) = C(\eta)$. Then

$$\alpha^2 = \frac{C_0 K \eta_{co} Re_{co}^{-m} \left[\frac{1}{n+1} - \frac{2}{n+2} + \frac{1}{n+3} \right]}{1-\eta_{co}}. \quad (27)$$

The constant of integration $C(\eta)$ can be determined on the basis of the following considerations. At the end of the cyclone where the outlet for the gases is located (i.e., at $\xi = 1$), the axial component of velocity at $1 > \eta > \eta_n$ is equal to zero. In the outlet opening itself, the motion of the gas can be represented as the superposition of two oppositely directed flows: the outgoing flow and the flow connected with infiltration. As a result of this superposition, a region occupied by an annular outgoing stream is formed in the outlet hole, while a region occupied by gas moving in the opposite direction is formed in the center. Having designated the boundary between these two regions as η_b , we conditionally assign the velocity profile in the outlet hole by means of the function

$$\omega_t = -A (\eta_n^K - \eta^K) (\eta_b^K - \eta^K). \quad (28)$$

To determine A and K , we use the balance equations:

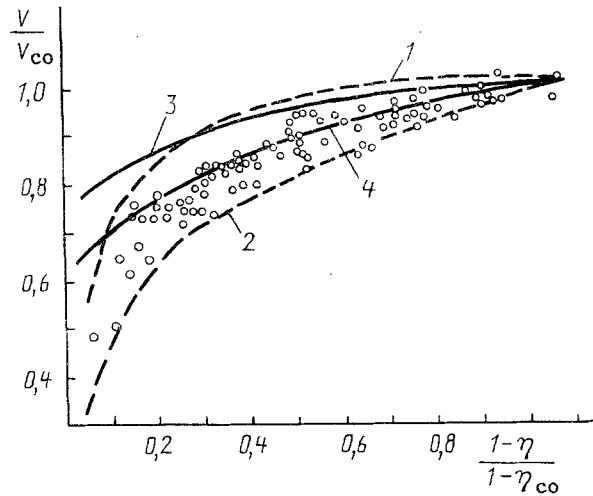


Fig. 2. Comparison of theoretical profiles of relative tangential velocity in the wall region of the cyclone (23) with experimental data from [8] and theoretical relations for a cyclone [9] and a plate [1]: 1) $\eta_{co} = 0.65$; 2) 0.885; 3) $\left(\frac{1-\eta}{1-\eta_{co}}\right)^{1/12}$ [9]; 4) $\left(\frac{1-\eta}{1-\eta_{co}}\right)^{1/7}$ [1].

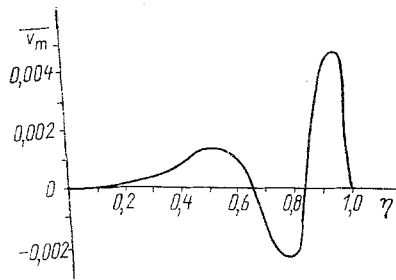


Fig. 3

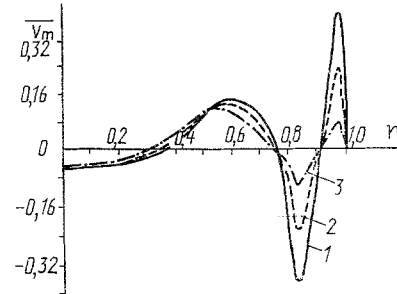


Fig. 4

Fig. 3. Theoretical profile of radial gas velocity.

Fig. 4. Theoretical profile of axial gas velocity: 1) $\xi = 0.2$; 2) 0.5; 3) 0.8.

$$-A2\pi R_c^2 \int_0^{\eta_n} \eta (\eta_n^K - \eta^K) (\eta_b^K - \eta^K) d\eta = G_g \quad (29)$$

$$-A2\pi R_c^2 \int_0^{\eta_b} \eta (\eta_n^K - \eta^K) (\eta_b^K - \eta^K) d\eta = -G_{rev} \quad (30)$$

where G_{rev} is the magnitude of the reverse axial stream. Having integrated and having divided (29) by (30), we obtain the equation

$$\frac{G_g}{G_{rev}} \left[\frac{1}{2} - \frac{1+a^{-K}}{K+2} + \frac{a^{-K}}{2(K+1)} \right] = -a^2 \left[\frac{1}{2} - \frac{1+a^K}{K+2} + \frac{a^K}{2(K+1)} \right],$$

where $a = \eta_n/\eta_b$ and G_{rev} must be determined experimentally.

Solving this transcendental equation with assigned a and G_{rev} , we find K and we determine A from (29). We then obtain

$$C(\eta) = \frac{\omega_{av} \tau}{v_m} = \frac{\omega_{av} a^K \left[1 - \left(\frac{\eta}{\eta_n} \right)^K \right] \left[\frac{1}{a^K} - \left(\frac{\eta}{\eta_n} \right)^K \right]}{v_m 2 \eta_n^2 \left[\frac{1}{2} - \frac{1 + a^K}{K + 2} + \frac{a^K}{2(K + 1)} \right]} \quad (31)$$

at $0 \leq \eta < \eta_n$.

$$C(\eta) = 0 \quad \text{at} \quad \eta_n \leq \eta \leq 1.$$

Figures 3 and 4 show the fields of the radial \bar{u} and axial \bar{w} components of gas velocity calculated from Eqs. (13-14), (25-26), and (31). The character of the distribution of \bar{u} and \bar{w} corresponds to the following pattern of gas flow in the core of a swirled flow in a cyclone chamber:

in the axial region, a central reverse flow ($\bar{w} = 0$) develops; this event is connected with the infiltration of gas through the outlet hole at the end of the cyclone and results in radial movement of gas from the center to the periphery ($\bar{u} > 0$):

an annular outgoing flow is formed in the middle of the cyclone chamber ($0.35 \leq \eta \leq 0.65$); here, gas travelling from the center and the periphery moves toward the outlet hole;

a powerful reverse annular flow is formed at the boundary of the core of the twisted flow; the radial component of velocity in this zone is equal to zero;

in the wall region, the gas moves toward the outlet hole.

The above-described distribution of the components of gas velocity in a cyclone agrees with literature data [4] on the character of flow in a cyclone. Additional empirical data will be necessary for a more accurate comparison. On the whole, above-constructed relations (12-14) and (23-24) can be used to calculate the tangential, axial, and radial components of gas velocity in a cyclone.

Notation. r, x, φ , radial, axial, and angular coordinates, m; u, w, v , radial, axial, and tangential components of velocity, m/sec; ρ , density of the gas, kg/m³; ν, ν_t , molecular and turbulent kinematic viscosities of the gas, m²/sec; R_c, L_c , radius and length of the cyclone, m; τ , shear stress, N/m²; η_{co} , relative radius of the boundary of the flow core; η_m , relative radius corresponding to the maximum value of tangential velocity; v_{co} , tangential velocity at the boundary of the flow core, m/sec; v_m , maximum value of tangential velocity of the gas, m/sec; G_g , gas flow rate, kg/sec.

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